ARE LARGE COMPLEX ECONOMIC SYSTEMS UNSTABLE ?

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Although classical economic theory is based on the concept of stable equilibrium, real economic systems appear to be always out of equilibrium. Indeed, they share many of the dynamical features of other complex systems, e.g., ecological foodwebs. We focus on the relation between increasing complexity of the economic network and its stability with respect to small perturbations in the dynamical variables associated with the constituent nodes. Inherent delays and multiple timescales suggest that economic systems will be more likely to exhibit instabilities as their complexity is increased even though the speed at which transactions are conducted has increased many-fold through technological developments. Analogous to the birth of nonlinear dynamics from Poincare's work on the question of whether the solar system is stable, we suggest that similar theoretical developments may arise from efforts by econophysicists to understand the mechanisms by which instabilities arise in the economy.

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Is the global economic system of the present era inherently unstable? It had long been thought that the cyclical sequence of inflations and recessions that have buffeted most national economies throughout the 19th and 20th centuries are an inevitable result of modern industrial capitalism. However, starting in the 1970s, economists allied with the influential Chicago school of economics started to promote the belief that the panacea to all economic ills of the world lay in completely and unconditionally subscribing to their particular brand of free-market policies. Their hubris reached its apogee at the beginning of this decade as summed up by the statement of the Nobel Laureate Robert Lucas at the 2003 annual meeting of the American Economic Association that "the central problem of depression prevention has been solved, for all practical purposes"[1]. This complacency about the economy's robustness to all possible perturbations led not only most professional economists, but more importantly, government bureaucrats and ministers (e.g., Gordon Brown's claims that economic booms and busts were a thing of the past[2]) to ignore or downplay the seriousness of the present economic and financial crisis at its initial stage. As many of the recent books on the onset of the global economic meltdown written by Posner and others point out, the mainstream economists and those whom they advised were blinded by their unquestioning acceptance of the assumptions of neo-classical theory[3].

In response to the rising criticism of traditional economic theory, spearheaded by physicists working on economic phenomena[4] as well as non-traditional economists who have collaborated with physicists[5], some economists are now trying to put up a defense that the sudden collapse of markets and banks is not something that can be predicted by economic theory as this contradicts their basic foundational principles of rational expectations and efficient markets. Thus, according to the conventional economic school of thought, bubbles cannot exist because any rise in price must reflect all information available about the underlying asset[6]. Although detailed analysis of data from markets clearly reveals that much of the observed price variation cannot be explained in terms of changes in

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economic fundamentals[7], the unquestioning belief in the perfection of markets has prompted several economists in the past decades to assert that the famous historical bubbles, such as Tulipomania in 17th century Holland or the South Sea Affair of 18th century England, were not episodes of price rise driven by irrational speculation, but rather were based on sound economic reasons[8]. This complete divorce of theory from observations points to the basic malaise of economics. What makes it all the more worrying is that despite the lack of any empirical verification, such economic theories have nevertheless been used to guide the policies of national and international agencies affecting the well-being of billions of human beings.

In fact, in its desperate effort to become a rigorous science by adopting, among other things, the formal mathematical framework of game theory, mainstream economics has become concerned less with describing reality than with an idealized version of the world. As an economist recently pointed out, in the overly mathematical formalism of rational expectations theory, any economic transaction, including that of a person buying a newspaper from the corner store vendor, appears to be a complicated chess game between Kenneth Arrow and Paul Samuelson (two of the most notable post-war economists)[9]. In truth, almost throughout our life, we rarely go through a complicated optimization process in an effort to calculate the best course of action. Even if we had access to complete information about all the options available (which is seldom the case), the complexity of the computational problem may overwhelm our decision-making capabilities. Thus, most often we are satisfied with choices that seem “good enough” to us, rather than the best one under all possible circumstances. Moreover, our choices may also reflect non-economic factors such as moral values that are usually not taken into consideration in mainstream economics.

Given these caveats, it seems that the cherished hypotheses of efficient markets and rational agents stands on very shaky ground indeed. The question obviously arises as to whether there are any alternative foundations that can replace the neo-classical framework. Behavioral economics, which tries to integrate the areas of psychology, sociology and economics, is one possible candidate. Another challenger is from outside the traditional boundaries of economics, a discipline that has been dubbed econophysics[10,12]. Although physicists have earlier worked on economic problems occasionally, it was only about a decade and half ago that a systematic, concerted movement began which has seen more and more physicists using the tools of their trade to analyze phenomena occurring in a socio-economic context[12]. This was partly driven by the availability of large quantities of high-quality data and the means to analyze them using computationally intensive algorithms. One of the most active sub-fields within this area is the empirical characterization of statistical properties of financial markets. Starting from the work of Mantegna and Stanley[13], several important results are now known about such markets which appear to be *universal*, in the sense that they are invariant with respect to the systems being considered, the time-period under consideration and the type of data being analyzed. One of the best examples of such universal features of financial markets is the *inverse cubic law* for the distribution of price (or index) fluctuations[14]. Not only has it been observed to hold across several different time-scales and across different types of stocks (and market indices), but more surprisingly, it appears to be valid irrespective of the stage of development of the market[15].

Financial markets have also proved a fertile ground for uncovering the structure of interactions between the different components of an economic system. In particular, the transactions between agents buying and selling different stocks in the market are reflected in the correlated movements of the prices of different stocks. Analogous to the process of inferring the movement of air molecules by watching the Brownian motion of suspended particles, we can have a coarse-grained view of the interaction dynamics between individuals in the market by reconstructing the network of significantly correlated stocks (i.e., correlated in terms of their price fluctuations). Comparison of such stock interaction networks for different markets has hinted that a financial market at a later stage of development possesses many more strongly bound clusters of co-moving stocks that are often from the same business sector[16]. As such markets tend to have identical statistical properties in terms of the distributions of price or index fluctuations, as well as, other trading indicators, they differ primarily in the topological structure of the interactions between their components. Thus, network analysis can provide us with a window into the process of economic development.

When we broaden our problem from the relatively restricted context of financial markets to general economic phenomena, the role played by networks of interactions become even more intriguing. Traditionally, economics has been concerned primarily with equilibria. Fig. 1 shows that the price mechanism was perceived by economists to introduce a negative feedback between perturbations in demand and supply, so that the system quickly settles to the equilibrium where supply exactly equals demand. Much of the pioneering work of Samuelson[17], Arrow[18] and others (for a review[19]) had been involved with demonstrating that such equilibria can be stable, subject to several restrictive conditions. However, the occurrence of complex networks of interactions in reality bring new dynamical issues to fore. Most notably, we are faced with the question: do complex

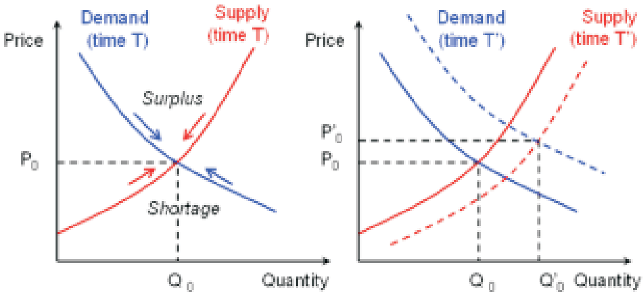

Fig. 1: Price mechanism leading to stable equilibrium between supply and demand according to traditional economic thinking. (Left) The supply and demand curves indicate how increasing supply or decreasing demand can result in falling price or vice versa. If the available supply of a certain good in the market at any given time is less than its demand for it among consumers, its price will go up. The perceived shortage will stimulate increase in production that will result in an enhanced supply. However, if supply increases beyond the point where it just balances the demand at that time, there will be unsold stock remaining which will eventually push the price down. This in turn will result in a decrease in the production. Thus, a negative feedback control mechanism governed by price will move demand and supply along their respective curves to the mutual point of intersection, where the quantity available (Q_0) at the equilibrium price P_0 is such that supply exactly equals demand. (Right) As the demand and supply of a product changes over time due to various different factors, the supply and demand curves may shift on the quantity-price space. As a result, the new equilibrium will be at a different price (P'_0) and quantity (Q'_0). Until the curves shift again, this equilibrium will be stable, i.e., any perturbation in demand or supply will quickly decay and the system will return to the equilibrium.

economic networks give rise to instabilities ? Given that most economic systems at present are composed of numerous strongly connected components, will periodic and chaotic behavior be the norm for such systems rather than static equilibrium solutions ?

This question has, of course, been asked earlier in different contexts. In ecology, it has given rise to the long-standing stability-diversity debate[20]. In the network framework, the ecosystem can be thought of as a network of species, each of the nodes being associated with a variable that corresponds to the population of the species it represents. The stability of the ecosystem is then defined by the rate at which small perturbations to the populations of various species decay with time. If the disturbance instead grows and gradually propagates through the system affecting other nodes, the equilibrium is clearly unstable. Prior to the pioneering work of May in the 1970s, it was thought that increasing complexity of an ecosystem, either in terms of a rise in the total number of species or the density and strength of their connections, results in enhanced stability of the ecosystem. This belief was based on empirical observations that more diverse foodwebs (e.g., in the wild) showed less violent fluctuations in population density than simpler communities (such as in fields under monoculture) and were less likely to suffer species extinctions. It has also been seen that tropical forests, which generally tend to be more diverse than sub-tropical ones, are also more resistant to invasion by foreign species[21]. It was therefore nothing short of a shock to the field when in 1972, Robert May showed in the very brief article *Will a large complex system be stable?*[22] using linear stability arguments that as complexity increases, a randomly connected network would tend to become more and more unstable.

The surprising demonstration that a system which has many elements and/or dense connections between its elements is actually more likely to suffer potentially damaging large fluctuations initiated by small perturbations immediately led to a large body of work on this problem[23] (for a review). The two major objections to May's results were (a) it uses linear stability analysis and that (b) it assumed random organization of the interaction structure. However, more recent work which consider systems with different types of population dynamics in the nodes, including periodic limit-cycles and chaotic attractors[24,25], as well as, networks having realistic features such as clustered small-world property[26] and scale-free degree distribution[27], have shown the result of increasing instability of complex networks to be extremely robust. While large complex networks can still arise as a result of gradual evolution[28], it is almost inevitable that such systems will be frequently subject to large fluctuations and extinctions.

The relevance of this body of work to understanding the dynamics of economic systems has been highlighted in the wake of the recent banking crisis when a series of defaults, following each other in a cascading process, led to the collapse of several major financial institutions. In fact, May and two other theoretical ecologists have written an article entitled *Ecology for bankers*[29] to point out the strong parallels between understanding collapse in economic and ecological networks. Recent empirical determination of networks occurring in the financial context, such as that of interbank payment flows between banks through the *Fedwire* real-time settlement service run by the US Federal Reserve, has now made it possible to analyze the process by which cascades of failure events can occur in such systems[30]. Analogous to ecological systems, where population fluctuations of a single species can trigger diverging deviations from the equilibrium in the populations of other species, congestion in settling the payment of one bank can cause other pending settlements to accumulate rapidly setting up the stage for a potential major failure event. It is intriguing that it is the very complexity of the network that has made it susceptible to such network propagated effects of local deviations which makes global or network-wide failure even more likely. As the world banking system becomes more and more connected, it may be very valuable to understand how the topology of interactions can affect the robustness of the network.

The economic relevance of the network stability arguments used in the ecological context can be illustrated from the following toy example. Consider a model financial market comprising N agents where each agent can either buy or sell at a given time instant. This tendency can be quantitatively measured by the probability to buy, p, and its complement, the probability to sell, $1 - p$. For the market to be in equilibrium, the demand should equal supply, so that as many agents are likely to buy as to sell, i.e., $p = 0:5$. Let us in addition consider that agents are influenced in their decision to buy or sell by the actions of other agents with whom they have interactions. In general, we can consider that out of all possible pairwise interactions between agents only a fraction C are actually realized. In other words, the inter-agent connections are characterized by the matrix of link strengths $\mathbf{J}=\{J_{ij}\}$ (where $i, j = 1, ..., N$ label the agents) with a fraction C of non-zero entries. If $J_{ij} > 0$, it implies that an action of agent j (buying or selling) is likely to influence agent i to act in the same manner, whereas $J_{ij} < 0$ suggests that the action of i will be contrary to that of j. Thus, the time-evolution of the probability for agent i to buy can be described by the following linearized equation close to the equilibrium $p_i = 0:5$ ($i = 1, ..., N$):

$$\frac{dp_i}{dt} = \varepsilon_i(0.5 - p_i) + \Sigma_j J_{ij}(0.5 - p_j), \qquad (1)$$

where ε_i is the rate of converge of an isolated node to its equilibrium state of equal probability for buying or selling. Without much loss of generality we can consider $\varepsilon_i = 1$ by appropriate choice of time units for the dynamics. If in addition, we consider that for simplicity the interactions are assigned randomly from a Gaussian distribution with mean 0 and variance σ^2, then the largest eigenvalue of the corresponding Jacobian matrix $\mathbf{J}$ evaluated around the equilibrium is $\lambda_{max} = \sqrt{NC\sigma^2 - 1}$. For system parameters such that $NC\sigma^2 > 1$, an initially small perturbation will gradually grow with time and drive the system away from its equilibrium state. Thus, even though the equilibrium $p = 0.5$ for individual nodes is stable in isolation, it may become unstable under certain conditions when interactions between the agents are introduced. Note that the argument can be easily generalized to the case where the distribution from which J_{ij} is chosen has a non-zero mean.

Another problem associated with the classical concept of economic equilibrium is the process by which the system approaches it. Walras, in his original formulation of how prices achieve their equilibrium value had envisioned the *tâtonnement* process by which a market-maker takes in buy/sell bids from all agents in market and gradually adjusts price until demand equals supply. Formally, it resembles an iterative convergence procedure for determining the fixed-point solution of a set of dynamical equations. However, as we know from the developments in nonlinear dynamics over the past few decades, such operations on even simple non-linear systems (e.g., the logistic equation) can result in periodic cycles or even chaos[31]. It is therefore not surprising to consider a situation in which the price mechanism can actually result in supply and demand to be forever out of step each other even though each is trying to respond to changes in the other. A simple situation in which such a scenario can occur is shown in Fig. 2, where a delay in the response of the supply to the changes in price through variations in demand can cause persistent oscillations.

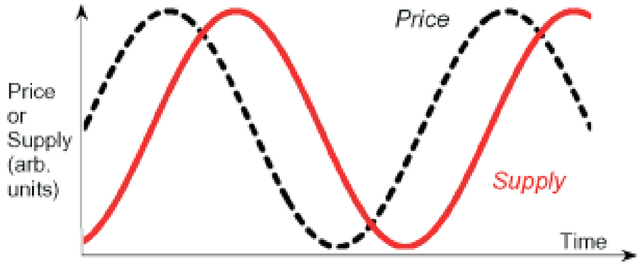

Fig. 2: Delay in market response can result in persistent price oscillations. Ideally the price mechanism should result in a transient increase (decrease) in demand to be immediately matched by a corresponding increase (decrease) in supply. However, in reality there is delay in the information about the rise or fall in demand reaching the producer; moreover, at the production end it may take time to respond to the increasing demand owing to inherent delays in the production system. Thus, the supply may always lag behind the price in a manner that produces oscillations: as price rises, supply initially remains low before finally increasing, by which time demand has fallen due to the high price which (in association with the increased supply) brings the price down. Supply continues to rise for some more time before starting to decrease. When it falls much lower than the demand, the price starts rising again which starts the whole cycle anew. Thus, if the demand fluctuates at a time-scale that is shorter than the delay involved in adjusting the production process to respond to variations in demand, the price may evolve in a periodic or even a chaotic manner.

As the principal reason for the instability appears to be the delay, one can argue that by increasing the speed of information propagation it should be possible to stabilize the equilibrium. However, we seem to have witnessed exactly the reverse with markets becoming more volatile as improvements in communication enable economic transactions to be conducted faster and faster. As a history of financial manias and panics points out, "there is little historical evidence to suggest that improvements in communications create docile financial markets . . . "[32]. A possible answer to this apparent paradox lies in the fact that in any realistic economic situation, information about fluctuations in the demand may require to be relayed through several intermediaries before it reaches the supplier. In other situations, the market may be segmented into several communities of agents, with significantly more interactions occurring between agents within the same as opposed to different communities. These features can introduce several levels of delays in the market, resulting in a multiple time-scale problem[33]. Thus, increasing the speed of transactions, while ostensibly allowing faster communication at the global scale can disrupt the dynamical separation between process operating at different time-scales. This can prevent sub-systems from converging to their respective equilibria before subjecting them to new perturbations, thereby always keeping the system out of equilibrium.

Therefore, we see that far from conforming to the neoclassical ideal of a stable equilibrium, the dynamics of the economic system is likely to be always far from equilibrium. In analogy with the question asked about ecological and other systems with many diverse interacting components, we can ask whether a sufficiently complex economy is bound to exhibit instabilities. After all, just like the neo-classical economists, natural scientists also at one time believed in the clockwork nature of the physical world (which in turn influenced the English philosopher, Thomas Hobbes, to seek the laws for social organization). However, Poincare's work on the question of whether the solar system is stable showed the inherent problems with such a viewpoint and eventually paved the way for the later developments of chaos theory. Possibly we are at the brink of a similar theoretical breakthrough in econophysics, one that does not strive to re-interpret (or even ignore) empirical data so as to conform to a theorist's expectations but one which describes the mechanisms by which economic systems actually evolve over time. It may turn out that, far from failures of the market that need to be avoided, crashes and depressions may be the necessary ingredients of future developments, as has been suggested by Schumpeter in his theory of *creative destruction*[34]. However, most importantly, we should not forget that economic phenomena form just one aspect of the entire set of processes that make up the human social organization. Econophysics has to ultimately strive to be a theory for the entire spectrum of human social behavior. As Keynes, one of the greatest economists, had once said "do not let us overestimate the importance of the economic problem, or sacrifice to its supposed necessities other matters of greater and more permanent significance"[35]. ❐

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